Analysis of Edge-Coupled Shielded Strip and Slabline Structures

STEWART M. PERLOW, SENIOR MEMBER, IEEE

Abstract —A method for the analysis of edge-coupled strip and slabline structures is presented which uses relatively simple algebraic expressions. The described procedure can be used to analyze directional couplers, interdigitated and comb line filters, and any other structures formed by an array of rectangular edge-coupled transmission lines, of arbitrary thickness, situated between two parallel plates. The analysis is complete in that it allows the conversion of the physical dimensions of the structure into a set of admittance parameters which completely describes its electrical behavior, including both conductor and dielectric losses.

I. Introduction

ANALYSIS OF shielded strip or slabline structures, such as directional couplers or interdigitated and comb line filters, requires a technique for converting the physical dimensions of the circuit into electrical parameters. Cohn has provided rigorous formulas relating the dimensions of zero-thickness coupled striplines to their odd- and even-mode impedances [1]. These solutions are based on successive transformations in a conformal mapping technique. He attempted to extend these results to thick strips by utilizing intuitive approximations for the odd- and even-mode fringing capacitance of the thick strips. These approximations work well over a limited range of strip thicknesses and strip separations.

Getsinger provided a rigorous solution for the thick-strip case by using an exact conformal mapping method [2]. However, his analytic results are given in terms of various elliptic functions which require a complicated process of evaluation of the functions themselves and the arguments of the functions. In order to make these results useful to the circuit designer, Getsinger provides accurate graphs which are used to extract the desired information.

Unfortunately, neither form of Getsinger's solution, analytic or graphical, is readily adaptable to computer-aided design algorithms. The analytic results require complicated, time-consuming algorithms for evaluation. The graphical results could be accommodated by storing data for each curve in memory and interpolating between data points for values that are not stored. Although this technique is acceptable in extreme circumstances, it certainly does not make efficient use of computer memory.

The technique developed in this discussion utilizes easily solvable analytic expressions, based upon both Cohn's and Getsinger's results. In addition, it extends these results by providing an expression for the effective width of very narrow strips. The effect of conductor and dielectric losses is also included. The expressions provide a method for achieving a very efficient algorithm for the analysis of edge-coupled strip and slabline structures which can be used on a computer or even on hand-held scientific calculators.

Manuscript received October 27, 1986; revised January 20, 1987.

The author is with the Microwave Technology Center, David Sarnoff Research Center, Princeton, NJ 08540-0432.

IEEE Log Number 8613830.

II. Characteristic Admittance

In considering the analysis of an array of coupled lines, the admittance matrix for the entire array can be formed if the odd- and even-mode admittances between adjacent lines along with the appropriate propagation constants are known.

Transmission-line theory shows that the relationship between the characteristic admittance of a lossless uniform transmission line, which is operated in the TEM mode, and the static capacitance per unit length is

$$Y_0 = vC = \frac{\sqrt{\varepsilon_r}}{\eta}\frac{C}{\varepsilon} \tag{1}$$

where

- v speed of propagation along the line,
- ε_r relative dielectric constant of the medium,
- ε permitivity of the dielectric medium,
- η impedance of free space (376.73 $\Omega/\square$).

In the case of two coupled transmission lines operating in the TEM mode, propagation along the lines can be broken into two independent or orthogonal modes. In the most general case, these two modes are referred to as the Π and C modes [3]. Since this discussion will be limited to a medium which is uniform or homogeneous, these two modes reduce to the well-known odd and even modes [1], [4], [5]. The characteristic admittance of the odd mode for each of the coupled strips is the same as the admittance for each of the strips if an electric wall (at the same potential as the upper and lower shielding planes) were placed between them. The characteristic admittance for the even mode for each of the strips is the same as the admittance of each strip if the electric wall were replaced by a magnetic wall.

The even-mode characteristic admittance of each line is related to the self-capacitance of the line (the capacitance between the line and ground), which is also known as the even-mode capacitance, by the relationship

$$Y_{0e1} = \frac{\sqrt{\varepsilon_r}}{\eta} \frac{C_{e1}}{\varepsilon} \tag{2a}$$

$$Y_{0e2} = \frac{\sqrt{\varepsilon_r}}{\eta} \frac{C_{e2}}{\varepsilon} \tag{2b}$$

where the subscripts refer to the first and second strips.

The odd-mode characteristic admittance of each line is related to the coupling capacitance between the lines and the self-capacitance of the lines by the relationships

$$Y_{0o12} = \frac{\sqrt{\varepsilon_r}}{\eta}\left(\frac{2C_c}{\varepsilon} + \frac{C_{e1}}{\varepsilon}\right) \tag{3a}$$

$$Y_{0o21} = \frac{\sqrt{\varepsilon_r}}{\eta}\left(\frac{2C_c}{\varepsilon} + \frac{C_{e2}}{\varepsilon}\right). \tag{3b}$$

It is easily seen that if the self-capacitances and the coupling capacitances are determined, the even- and odd-mode admittances are known.

III. The Coupling Capacitance

Fig. 1(a) provides a representation of the physical dimensions of a coupled line array. The capacitive components of the lines are shown in Fig. 1(b). It will be assumed in this discussion that the thicknesses of all the strips in the array are the same and that the coupling between nonadjacent strips is negligible.

The coupling capacitance per unit length of the lines, C_c, can be calculated using the exact analytic equations for the even- and odd-mode fringing capacitances given by Cohn for zero-thickness lines, and compensating for line thickness by the value A as shown in (4), (5), and (6) below. The capacitance per unit length for a strip of thickness t, which is a function of the strip separation s, and ground-plane spacing b is

$$\frac{C_c}{\varepsilon} = A\left[\frac{C_c}{\varepsilon}\right]_{t=0} \tag{4}$$

where the coupling capacitance per unit length for zero-thickness strips is

$$\left[\frac{C_c}{\varepsilon}\right]_{t=0} = \frac{2}{\pi}\ln\left[\coth\left(\frac{\pi}{2}\frac{s}{b}\right)\right] \tag{5}$$

and

$$A = \begin{cases} 1.0 + 2.13507\dfrac{t}{b}\left(\dfrac{s}{b}\right)^{-0.57518} & \dfrac{s}{b} < 0.3 \\ 1.0 + 3.89531\dfrac{t}{b}\left(\dfrac{s}{b}\right)^{-0.11467} & \dfrac{s}{b} \geq 0.3. \end{cases} \tag{6}$$

The thick strip correction factor A was derived from Getsinger's results [2] by a judicious choice of normalization factors.

IV. Fringing Capacitances

The fringing capacitances represent the nonuniform field regions on the edges and corners of the strips. In this discussion, two types of fringing capacitances will be considered. The outside edges of the first and the last strip have the same fringing capacitance that a single isolated strip would have. This will be called the external fringing capacitance; it is denoted by C_f and is a function only of the strip's thickness and the separation between ground planes. The second type occurs on the interior edges of the strips and is denoted by C_{fe}. This interior fringing capacitance, also known as the even-mode fringing capacitance, is a function of strip thickness, ground-plane separation, and spacing between adjacent strips.

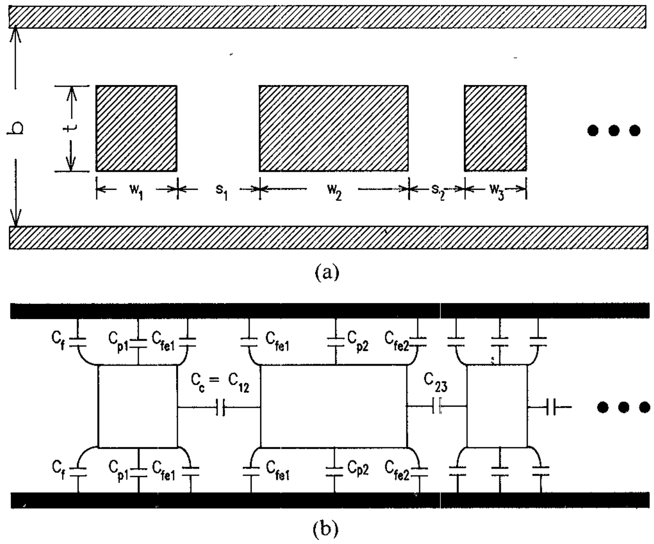

Fig. 1. (a) Physical structure of coupled line array. (b) Capacitive components of the array.

The external fringing capacitance C_f can be determined by subtracting the parallel-plate capacitance from the total capacitance of a single isolated strip between ground planes [6] or by using the exact equation for a semi-infinite strip given by Cohn [7]:

$$\frac{C_f}{\varepsilon} = \frac{1}{\pi}\left[2t_b\ln(t_b+1) - (t_b-1)\ln(t_b^2-1)\right] \tag{7}$$

where

$$t_b = \frac{1}{\left(1 - \dfrac{t}{b}\right)}.$$

The internal, or even-mode, fringing capacitance C_{fe}, like the coupling capacitance, is a function of the strip thickness, the separation of the ground planes, and the spacing between the strips. It can be calculated from

$$\frac{C_{fe}}{\varepsilon} = A\left\{\frac{s}{b} - \frac{2}{\pi}\ln\left[\cosh\left(\frac{\pi}{2}\frac{s}{b}\right)\right]\right\} + B \tag{8}$$

where A is the value defined in (6) and B is defined as follows.

For $t/b \leq 0.5636$,

$$B = \begin{cases} -\left(0.2933 + 3.333\dfrac{s}{b}\right)\dfrac{t}{b} & \dfrac{s}{b} < 0.08 \\ -0.56\dfrac{t}{b} & \dfrac{s}{b} \geq 0.08. \end{cases} \tag{9}$$

For $t/b > 0.5636$,

$$B = \begin{cases} -0.1653 - 5.6814\frac{s}{b} + 6.7475\frac{s}{b}\frac{t}{b} & \frac{s}{b} < 0.08 \\ -0.62 + 0.54\frac{t}{b} & \frac{s}{b} \geq 0.08. \end{cases} \tag{10}$$

The coefficient of the A term in (8) is Cohn's formula for the even-mode fringing capacitance for the zero-thickness case. The equations determining the value of B were generated from Getsinger's data.

V. Parallel-Plate Capacitance

The fringing capacitances derived by Cohn and Getsinger were obtained assuming that each strip was of semi-infinite width. This assumption was required to eliminate the possibility of any interaction between the fringing fields on both edges of the strip. If the width of any of the strips is small enough, the distortion of the fields at the edges of the strip can extend across the entire width of the strip, eliminating the parallel-plate region completely. Getsinger approached this problem by defining an effective width w_{eff}, which was related to the physical width by the relationship $w_{\text{eff}} = 1.2w - 0.07(b-t)$. This relationship was to be used if $w < 0.35(b-t)$; it is valid if $0.1 < w_{\text{eff}}/(b-t) < 0.35$. This formula is based on a linear approximation to the exact fringing capacitance of a single strip of zero thickness. Riblet has pointed out that this criterion, based on zero-thickness strip, is too cautious for strips with thickness [8].

A better approximation for the effective width can be obtained by subtracting the fringing capacitances from the total capacitance of an isolated thick strip. The resulting capacitance is the effective parallel-plate capacitance per unit length. The effective width of the parllel-plate or uniform field region is then calculated from the effective capacitance. Fig. 2 shows the relationship between the physical width and the effective width for several values of thickness, along with the results using Getsinger's linear approximation. For zero-thickness strips, the linear approximation is very accurate in the region $0.142 < w/b < 0.27$. Above $w/b = 0.27$, the linear approximation increases a little too rapidly. As the thickness increases, the discrepancy between the two approximations increases. The linear approximation indicates that the effective width is not the same as the physical width when $t/b = 0.2$ and w/b is less than 0.28. The thick-strip approximation indicates that both widths are the same for a value of w/b as small as 0.1 and are very nearly equal for values of w/b as small as 0.05. For $t/b = 0.4$, the effective width and the physical width are the same for values of $w/b > 0.04$. The linear approximation indicates that this condition is not met until w/b is at least 0.21.

The following equation relating the physical width w to the effective width w_{eff} was obtained by using [6] to calculate the total capacitance of a thick strip of any width:

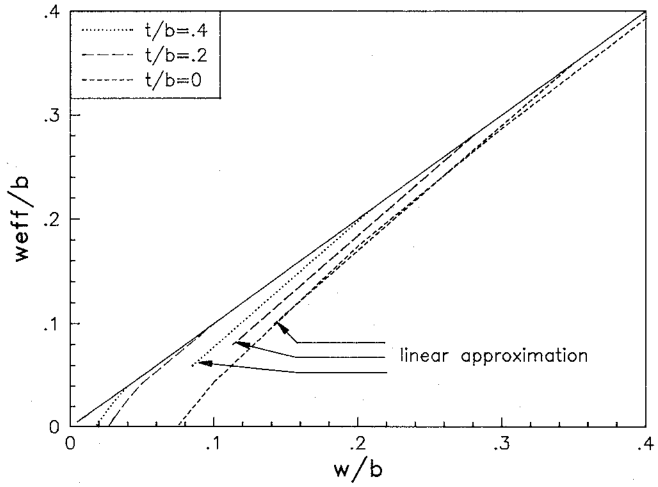

Fig. 2. Relationship between the effective and physical widths of finite-width strips of various thicknesses (calculated using (11)) along with Getsinger's linear approximation.

$$\frac{w_{\text{eff}}}{b} = \begin{cases} \frac{w}{b} - W_{c0}\left[1.0 - \sqrt{\frac{\frac{w}{b}\frac{t}{b}}{0.015}}\right]^2 & \frac{w}{b}\frac{t}{b} < 0.015 \\ \frac{w}{b} & \frac{w}{b}\frac{t}{b} \geq 0.015 \end{cases} \tag{11}$$

where

$$W_{c0} = \frac{2}{\pi}\ln 2 + \frac{w}{b} - \frac{1}{\frac{2}{\pi}\ln\left[2\coth\left(\frac{\pi}{4}\frac{w}{b}\right)\right]} \tag{12}$$

is the correction for zero-thickness strips.

Examination of the correction term indicates that the difference between the effective width and the physical width is negligibly small if

$$\frac{w}{b} > 0.4 \tag{13a}$$

or

$$\frac{w}{b} > \frac{0.015}{\frac{t}{b}}. \tag{13b}$$

The parallel-plate capacitance from the strip to one of the grounding planes, C_p, is calculated using the effective width:

$$\frac{C_p}{\varepsilon} = 2\frac{w_{\text{eff}}}{b-t}. \tag{14}$$

VI. The Total Self- or Even-Mode Capacitance

The total self-capacitance (the even-mode capacitance) for each strip of the array can be calculated as the sum of all the component capacitances, as shown in Fig. 1(b).

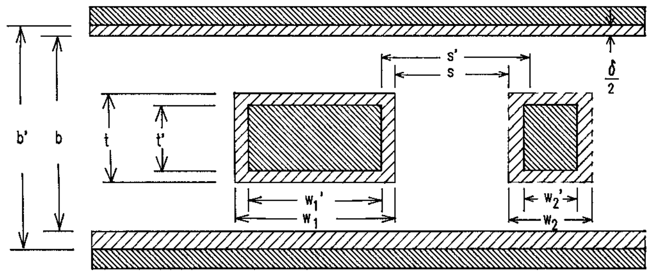

Fig. 3. Reduction of physical dimensions by the skin depth.

The first and the last strip of the array each have an external fringing capacitance, a parallel-plate capacitance, and an internal or even-mode fringing capacitance component. Therefore, the total self-capacitance is

$$C_1 = 2(C_f + C_{p1} + C_{fe1}) \tag{15}$$

$$C_N = 2(C_f + C_{pN} + C_{feN}) \tag{16}$$

where the subscript 1 refers to the first line, and subscript N to the last line.

Each of the interior lines has two even-mode fringing capacitance components and a parallel-plate component:

$$C_k = 2(C_{fe(k-1)} + C_{pk} + C_{fek}) \tag{17}$$

where the subscript k refers to the strip; $C_{fe(k-1)}$ is the even-mode fringing of the strip on the edge closest to the previous strip; and C_{fek} is the even-mode fringing capacitance on the edge closest to the next strip. Note that the external fringing capacitance C_f is the same for the first and the last strip since it is implicitly assumed that all strips are of the same thickness.

VII. Line Losses

The losses of the coupled line array are characterized by using the attenuation constant α, which is the real part of the propagation constant. The calculation of the component of α due to conductor losses is accomplished by using Wheeler's incremental inductance rule and numerical differentiation [9], [10]. The process is very simple. The skin depth δ is calculated using the relative resistivity of the metal being used as

$$\delta = 0.0822\sqrt{\frac{\rho_r}{f}} \tag{18}$$

where ρ_r is the resistivity of the metal relative to copper, f is the frequency in gigahertz, and δ is the skin depth in mils.

The coupling capacitances and self-capacitances are calculated using the physical dimensions. They are then recalculated using the new dimensions which would occur if each metal surface were reduced by the value of $\delta/2$, as shown in Fig. 3. The new dimensions become

$$\begin{aligned} b' &= b + \delta \\ t' &= t - \delta \\ w_k' &= w_k - \delta \\ s_k' &= s_k + \delta \end{aligned} \tag{19}$$

where subscript k refers to a particular strip.

The attenuation constants can then be calculated using the original values of capacitance, C_k and $C_{k,(k+1)}$, and the new values calculated using the dimensions corrected by the skin depth, C_k' and $C_{k,(k+1)}'$ (the subscripts $k,(k+1)$ referring to the coupling capacitance between strips k and $k+1$):

$$\alpha_{k,(k+1)} = \pi\sqrt{\varepsilon_r}\,\frac{f}{0.2998}\left[1-\left(\frac{C_k' + 2C_{k,(k+1)}'}{C_k + 2C_{k,(k+1)}}\right)\right] \text{(Np/m)} \tag{20}$$

$$\alpha_{(k+1),k} = \pi\sqrt{\varepsilon_r}\,\frac{f}{0.2998}\left[1-\left(\frac{C_{(k+1)}' + 2C_{k,(k+1)}'}{C_{(k+1)} + 2C_{k,(k+1)}}\right)\right] \text{(Np/m)} \tag{21}$$

$$\alpha_k = \pi\sqrt{\varepsilon_r}\,\frac{f}{0.2998}\left[1-\left(\frac{C_k'}{C_k}\right)\right] \text{(Np/m)}. \tag{22}$$

The attenuation constants with a single subscript are the even-mode attenuation constants and those with the double subscript are the odd-mode attenuation constants. Note that between every pair of lines there are two different attenuation constants. In general, they have different values; to be equal the even-mode capacitances must be equal.

The component of the attenuation constant due to losses in the dielectric is calculated as

$$\alpha_d = \pi\sqrt{\varepsilon_r}\,\frac{f}{0.2998}\tan(\delta_d)\ \text{(Np/m)} \tag{23}$$

where $\tan\delta_d$ is the loss tangent of the dielectric.

In addition to specifying the attenuation constant, the imaginary part of the propagation constant, β, is also required:

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi\sqrt{\varepsilon_r}}{\lambda_0} \tag{24}$$

where

$$\lambda_0 = \frac{11.808}{f}\ \text{(in)} = \frac{29.98}{f}\ \text{(cm)}.$$

The propagation constants are

$$\begin{aligned} \gamma_k &= (\alpha_k + \alpha_d) + j\beta \\ \gamma_{k,(k+1)} &= (\alpha_{k,(k+1)} + \alpha_d) + j\beta \\ \gamma_{(k+1),k} &= (\alpha_{(k+1),k} + \alpha_d) + j\beta. \end{aligned} \tag{25}$$

VIII. The Admittance Matrix

The admittance matrix for the coupled line array completely describes the network. The elements of the admittance matrix are functions of the odd- and even-mode admittances [11]. The odd- and even-mode admittances were described in (2) and (3) in terms of the coupling capacitances and the self-capacitances. The general form of the mode admittances of the k and the $k+1$ line of the array is

even mode:

$$Y_{0ek} = \frac{\sqrt{\varepsilon_r}}{\eta}\frac{C_k}{\varepsilon} \tag{26a}$$

$$Y_{0e(k+1)} = \frac{\sqrt{\varepsilon_r}}{\eta}\frac{C_{(k+1)}}{\varepsilon} \tag{26b}$$

odd mode:

$$Y_{0ok,(k+1)} = \frac{\sqrt{\varepsilon_r}}{\eta}\left(\frac{2C_{(k,k+1)}}{\varepsilon} + \frac{C_k}{\varepsilon}\right) \tag{27a}$$

$$Y_{0o(k+1),k} = \frac{\sqrt{\varepsilon_r}}{\eta}\left(\frac{2C_{(k,k+1)}}{\varepsilon} + \frac{C_{(k+1)}}{\varepsilon}\right). \tag{27b}$$

If the nodes of the network are arranged as shown in Fig. 4, where nodes 1 and 2 are at the ends of strip 1, nodes 3 and 4 are at the ends of strip 2, and in general the nodes $2k-1$ and $2k$ are at the ends of strip k, then the admittance matrix of the coupled lines is of the form

$$\begin{matrix} y_{11} & y_{12} & \cdots & y_{1,m} & \cdots & y_{1,2N} \\ y_{21} & y_{22} & \cdots & y_{2,m} & \cdots & y_{2,2N} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ y_{m1} & y_{m2} & \cdots & y_{m,m} & \cdots & y_{m,2N} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ y_{2N,1} & y_{2N,2} & \cdots & y_{2N,m} & \cdots & y_{2N,2N} \end{matrix}$$

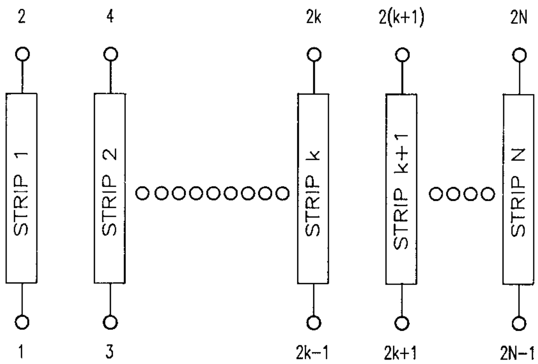

Fig. 4. Numbering of nodes of the coupled line array.

where the subscripts refer to the nodes at the ends of the strips, and N is the number of strips.

The admittance matrix can be filled in the standard way, i.e.,

1) The self-admittance of node m is the sum of all the admittances connected to node m.

2) The mutual admittance between node m and $m+1$ is the negative of the admittance connected directly between nodes m and $m+1$.

The self- and mutual admittances for any two adjacent lines in the coupled line array can be expressed in terms of their even- and odd-mode admittances. In general, if strip k is connected between nodes $2k-1$ and $2k$, and strip $k+1$ is connected between nodes $2k+1$ ($2(k+1)-1=2k+1$) and $2(k+1)$, the self-admittances at nodes $2k-1$, $2k$, $2k+1$, and $2k+2$ are

$$Y_{(2k-1),(2k-1)} = \begin{cases} \dfrac{Y_{0ek}}{2}\coth(\gamma_k l) + \dfrac{Y_{0ok,(k+1)}}{2}\coth(\gamma_{k,(k+1)} l) & k=1 \\ \dfrac{Y_{0ok,(k+1)}}{2}\coth(\gamma_{k,(k+1)} l) & k\neq 1 \end{cases} \tag{28}$$

$$Y_{2k,2k} = Y_{(2k-1),(2k-1)} \tag{29}$$

$$Y_{(2k+1),(2k+1)} = \begin{cases} \dfrac{Y_{0e(k+1)}}{2}\coth(\gamma_{(k+1)} l) + \dfrac{Y_{0o(k+1),k}}{2}\coth(\gamma_{(k+1),k} l) & k=N-1 \\ \dfrac{Y_{0o(k+1),k}}{2}\coth(\gamma_{(k+1),k} l) & k\neq N-1 \end{cases} \tag{30}$$

$$Y_{(2k+2),(2k+2)} = Y_{(2k+1),(2k+1)}. \tag{31}$$

The mutual admittances for the nodes $2k-1$ and $2k$, on strip k, are

$$Y_{(2k-1),2k} = \begin{cases} -\dfrac{Y_{0ek}}{2}\operatorname{csch}(\gamma_k l) - \dfrac{Y_{0ok,(k+1)}}{2}\operatorname{csch}(\gamma_{k,(k+1)} l) & k=1 \\ -\dfrac{Y_{0ok,(k+1)}}{2}\operatorname{csch}(\gamma_{k,(k+1)} l) & k\neq 1 \end{cases} \tag{32}$$

$$Y_{2k,(2k-1)} = Y_{(2k-1),2k}. \tag{33}$$

The mutual admittances for the nodes on strip $k+1$, $2k+1$, and $2k+2$ are

$$Y_{(2k+1),(2k+2)} = \begin{cases} -\dfrac{Y_{0e(k+1)}}{2}\operatorname{csch}(\gamma_{(k+1)} l) - \dfrac{Y_{0o(k+1),k}}{2}\operatorname{csch}(\gamma_{(k+1),k} l) & k=N-1 \\ -\dfrac{Y_{0o(k+1),k}}{2}\operatorname{csch}(\gamma_{(k+1),k} l) & k\neq N-1 \end{cases} \tag{34}$$

$$Y_{(2k+2),(2k+1)} = Y_{(2k+1),(2k+2)}. \tag{35}$$

The mutual admittances for the nodes $2k-1$ and $2k$, on strip k, with respect to the nodes on strip $k+1$ are

$$Y_{(2k-1),(2k+1)} = \frac{Y_{0ek}}{2}\coth(\gamma_k l) - \frac{Y_{0ok,(k+1)}}{2}\coth(\gamma_{k,(k+1)} l) \quad (36)$$

$$Y_{(2k-1),(2k+2)} = -\frac{Y_{0ek}}{2}\operatorname{csch}(\gamma_k l) + \frac{Y_{0ok,(k+1)}}{2}\operatorname{csch}(\gamma_{k,(k+1)} l) \quad (37)$$

$$Y_{(2k,2k+1)} = Y_{(2k-1),(2k+2)} \quad (38)$$

$$Y_{2k,(2k+2)} = Y_{(2k-1),(2k+1)}. \quad (39)$$

The mutual admittances for the nodes $2k+1$ and $2k+2$, on strip $k+1$, with respect to the nodes on strip k are

$$Y_{(2k+1),(2k-1)} = \frac{Y_{0e(k+1)}}{2}\coth(\gamma_{(k+1)} l) - \frac{Y_{0o(k+1),k}}{2}\coth(\gamma_{(k+1),k} l) \quad (40)$$

$$Y_{(2k+1),2k} = -\frac{Y_{0e(k+1)}}{2}\operatorname{csch}(\gamma_{(k+1)} l) + \frac{Y_{0o(k+1),k}}{2}\operatorname{csch}(\gamma_{(k+1),k} l) \quad (41)$$

$$Y_{(2k+2),(2k-1)} = Y_{(2k+1),2k} \quad (42)$$

$$Y_{(2k+2),2k} = Y_{(2k+1),(2k-1)}. \quad (43)$$

The analysis is performed by evaluating the admittances defined by (28)–(43) for values of k from 1 (the first line) to $N-1$, where N is the last line.

The admittance matrix defined in this manner completely characterizes the array of coupled lines, since any voltage or current at any of the nodes can be obtained by reduction of this matrix.

IX. Example—Analysis of a Coupler Formed by Two Unequal Width Lines

Cristal described a nonsymmetrical 10-db directional coupler, and this will be used as an example of the use of the equations shown above [11]. The coupler is meant to provide 10 dB of coupling into a 75-Ω termination while the through-ports are to be terminated in 50 Ω. The schematic arrangement is shown in Fig. 5. The physical dimensions are

$$b = 625 \text{ mils}$$
$$t = 250 \text{ mils}$$
$$\frac{t}{b} = 0.400$$
$$\frac{w_1}{b} = 0.508$$
$$\frac{w_2}{b} = 0.111$$
$$\frac{s}{b} = 0.233.$$

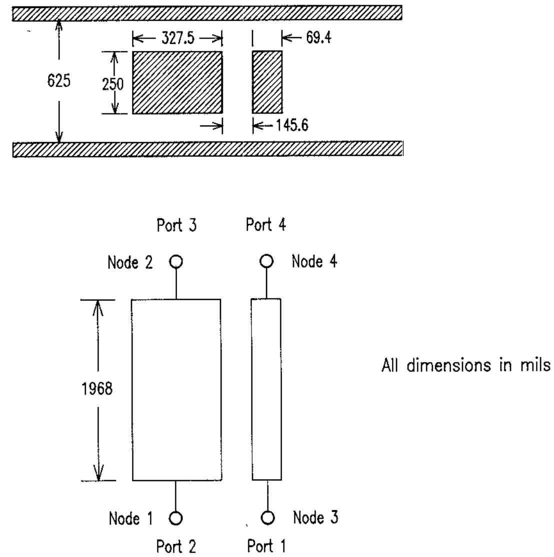

Fig. 5. Layout of a nonsymmetrical 10-dB coupler.

The coupling capacitance is determined using (4), (5), and (6):

$$A = 2.97404$$
$$\left[\frac{C_c}{\varepsilon}\right]_{t=0} = 0.66746$$
$$\frac{C_c}{\varepsilon} = 1.98505.$$

The external fringing capacitance is calculated using (7):

$$\frac{C_f}{\varepsilon} = 0.91860$$

and the internal fringing is calculated using (8) and (9):

$$B = -0.22400$$
$$\frac{C_{fe1}}{\varepsilon} = \frac{C_{fe2}}{\varepsilon} = 0.34488.$$

The parallel-plate capacitances are determined using (14). The effective width for either strip is the physical width of the strip since the condition given in (13b) is true:

$$\frac{C_{p1}}{\varepsilon} = 1.69333$$
$$\frac{C_{p2}}{\varepsilon} = 0.3700.$$

Since there are only two strips, (15) and (16) provide the self-capacitances:

$$\frac{C_1}{\varepsilon} = 5.91361$$
$$\frac{C_2}{\varepsilon} = 3.26695.$$

The values of even-mode and coupling capacitances should be compared with those used by Cristal in obtaining the required physical dimensions from Getsinger's graphs:

Capacitance	*This method*	*Cristal*
$\frac{C_c}{\varepsilon}$	1.985	2.050
$\frac{C_1}{\varepsilon}$	5.914	5.891
$\frac{C_2}{\varepsilon}$	3.267	3.244.

If it assumed that the material being used is copper, the skin depth at 1.5 GHz can be calculated:

$$\delta = 0.06712 \text{ mils.}$$

The new dimensions needed to obtain the new values of capacitance that are to be used in the calculation of the losses are

$$b' = b + \delta = 625.067 \text{ mils}$$
$$t' = t - \delta = 249.933 \text{ mils}$$
$$w_1' = w_1 - \delta = 317.433 \text{ mils}$$
$$w_2' = w_2 - \delta = 69.308 \text{ mils}$$
$$s' = s + \delta = 145.692 \text{ mils.}$$

These new dimensions are used in the same manner as the original dimensions to obtain the self- and coupling capacitances:

$$\frac{C_{12}'}{\varepsilon} = 1.98368$$
$$\frac{C_1'}{\varepsilon} = 5.91137$$
$$\frac{C_2'}{\varepsilon} = 3.26565.$$

The attenuation factors are calculated using (20), (21), and (22):

$$\alpha_{12} = .00795 \text{ Np/m}$$
$$\alpha_{21} = .00879 \text{ Np/m}$$
$$\alpha_1 = .00597 \text{ Np/m}$$
$$\alpha_2 = .00625 \text{ Np/m.}$$

The even-mode admittances for strips 1 and 2 are calculated using (26):

$$Y_{0e1} = 0.01570 \text{ S}$$
$$Y_{0e2} = 0.00867 \text{ S.}$$

The odd-mode admittances are obtained from (27):

$$Y_{0o12} = 0.02624 \text{ S}$$
$$Y_{0o21} = 0.01921 \text{ S.}$$

With the nodes as shown in Fig. 5, nodes 1 and 2 are at the ends of line 1, and nodes 3 and 4 are at the ends of line 2; the elements of the admittance matrix are

$$Y_{11} = \tfrac{1}{2}[.01570\coth[(.00597 + j\beta)l] + .02624\coth[(.00795 + j\beta)l]]$$
$$Y_{12} = \tfrac{1}{2}[-.01570\operatorname{csch}[(.00597 + j\beta)l] - .02642\operatorname{csch}[(.00795 + j\beta)l]]$$
$$Y_{13} = \tfrac{1}{2}[.01570\coth[(.00597 + j\beta)l] - .02624\operatorname{csch}[(.00795 + j\beta)l]]$$
$$Y_{14} = \tfrac{1}{2}[-.01570\operatorname{csch}[(.00597 + j\beta)l] + .02624\operatorname{csch}[(.00795 + j\beta)l]]$$
$$Y_{21} = Y_{12}$$
$$Y_{22} = Y_{11}$$
$$Y_{23} = Y_{14}$$
$$Y_{24} = Y_{13}$$
$$Y_{31} = \tfrac{1}{2}[.00867\coth[(.00625 + j\beta)l] - .01921\coth[(.00879 + j\beta)l]]$$
$$Y_{32} = \tfrac{1}{2}[-.00867\operatorname{csch}[(.00625 + j\beta)l] + .01921\operatorname{csch}[(.00879 + j\beta)l]]$$
$$Y_{33} = \tfrac{1}{2}[.00867\coth[(.00625 + j\beta)l] + .01921\coth[(.00879 + j\beta)l]]$$
$$Y_{34} = \tfrac{1}{2}[-.00867\operatorname{csch}[(.00625 + j\beta)l] - .1921\operatorname{csch}[(.00879 + j\beta)l]]$$
$$Y_{41} = Y_{32}$$
$$Y_{42} = Y_{31}$$
$$Y_{43} = Y_{34}$$
$$Y_{44} = Y_{33}.$$

This essentially completes the analysis since the 4-port Y-parameters are completely specified.

X. Conclusions

A procedure for analyzing edge-coupled slab and stripline arrays has been presented. Since it utilizes simple analytic expressions to replace graphical interpretation or the more complex evaluation of elliptic integrals and their arguments, it is ideally suited for computer-aided analysis and optimization. The analysis is performed by completely transforming the physical attributes of the coupled line array into Y-parameters which include the effects of loss in the metal and dielectric material.

References

[1] S.B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans. Microwave Theory Tech.* vol. MTT-3, no. 5, pp. 29–38, Oct. 1955.

[2] W.J. Getsinger, "Coupled rectangular bars between parallel plates," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, no. 1, pp. 65–72, Jan. 1962.

[3] V.K. Tripathi, "Asymmetric coupled transmission lines in an inhomogeneous medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 734–739, Sept. 1975.
[4] E.M.T. Jones and J.T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75–81, Apr. 1956.
[5] S.M. Perlow and A. Presser, "The interdigitated three-strip coupler," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1418–1422, Oct. 1984.
[6] S.M. Perlow, "Simplified expressions for the calculation of the impedance of shielded slab line of any shape ratio," *Microwave J.*, vol. 29, no. 11, pp. 125–131, Nov. 1986.
[7] S.B. Cohn, "Problems in strip transmission lines," *IRE Trans. Microwave Theory Tech.* vol. MTT-3, no. 2, pp. 119–126, Mar. 1955.
[8] H.J. Riblet, "The exact dimensions of a family of rectangular coaxial lines with given impedance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 538–541, Aug. 1972.
[9] H.A. Wheeler, "Formulas for the skin effect," *Proc. IRE*, vol. 30, pp. 412–424, Sept. 1942.
[10] H.A. Wheeler, "Transmission-line properties of a strip on a dielectric sheet on a plane," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 631–647, Aug. 1977.
[11] E.G. Cristal, "Coupled-transmission-line directional couplers with coupled lines of unequal characteristic impedances," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 337–346, July 1966.

Stewart M. Perlow (S'60–M'73–SM'83) received the B.E.E. degree from the City University of New York in 1963, and the M.S.E.E. degree in 1970 from the Polytechnic Institute of New York.

He joined RCA at the Advanced Communications Laboratories, New York, in 1963. For the next three years, he worked in the Microwave Circuits group on low-frequency active-filter techniques, parametric upconverters, parametric amplifiers, and UHF solid-state power amplifiers. He was cited for achievements as an outstanding young engineer and was used as an example in RCA's college recruitment literature. In 1966, he became a cofounder of National Electronic Laboratories, which was purchased by Harvard Industries in 1967. When Harvard acquired KMC Semiconductor in 1968, Mr. Perlow became Chief Engineer of KMC. His responsibilities included directing the evaluation, specification, and testing of new semiconductor devices and the design and manufacture of ultra-low-noise and broad-band solid-state amplifiers and receiving systems covering the complete RF spectrum. Since joining RCA Laboratories, Princeton, NJ, as a Member of the Technical Staff in 1973, Mr. Perlow has been involved in TV tuner design, nonlinear distortion analysis of RF signal processors, and computed-aided design and measurement.

Mr. Perlow holds several U.S. patents. He has received two RCA Laboratories Outstanding Achievement Awards, in 1980 and 1982, and a Software Recognition Award in 1986. He is a member of Eta Kappa Nu.